

INFO216: Knowledge Graphs

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Session S14: OWL DL

- Themes:
 - description logic
 - decision problems
 - OWL DL
 - Manchester OWL-syntax



Readings

- Forum links (cursory):
 - <http://www.w3.org/TR/owl2-primer/>
 - show: Turtle and Manchester syntax
 - hide: other syntaxes
 - Description Logic Handbook:
 - Chapter 1: Nardi & Brachman:
Introduction to Description Logics
 - Chapter 2: Baader & Nutt:
Formal Description Logics (*gets hard*)



Description Logic (DL)



Description logics

- Description Logic (DL)
 - a simple *fragment* of predicate logic
 - ...or, rather, a *family of such fragments*
 - not very *expressive* (“uttrykkskraftig”)
 - but (can have) *good decision problems*, i.e.,
 - it answers *decision problems* (rather) quickly
- Suitable for describing *concepts* (“begreper”)
 - formal basis for *OWL DL*
 - can be used to:
 - describe *concepts* and their *roles* (“Tbox”)
 - describe *individuals* and their *roles* (“ABox”)



Relationship to other logics

- *Proposition logics* are about *statements (propositions)*:
 "Martha is a Woman" \Leftarrow
 "Martha is Human" \wedge "Martha is Female"
- (First order) *predicate logics* are about *predicates* and *objects*:
 - $\forall x. (\text{Woman}(x) \Leftrightarrow \text{Human}(x) \wedge \text{Female}(x))$
- *Description logics* are about *concepts*:
 - **Woman** \doteq **Human** \sqcap **Female**
 - ...and also about *roles* and *individuals*
- There are many other logic systems:
 - *modal logics*: necessarily \square , possibly \diamond
 - *temporal logics*: always \square , sometimes \diamond , next time \circ



Definition of concepts (“begreper”)

- **Woman** \doteq **Human** \sqcap **Female**
- **Man** \doteq **Human** \sqcap \neg **Woman**
- **Parent** \doteq **Mother** \sqcup **Father**
 - concepts: **Human, Female, Woman...**
 - definition: \doteq
 - conjunction (and): \sqcap
 - disjunction (or): \sqcup
 - negation (not): \neg
 - nested expressions: ()
- **Childless** \doteq ..using **Human** and **Parent**..



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- **Childless** \doteq **Human** \sqcap \neg **Parent**



Types of concepts (“begreper”)

- **Woman** \doteq **Human** \sqcap **Female**
- **Man** \doteq **Human** \sqcap \neg **Woman**
- **Parent** \doteq **Mother** \sqcup **Father**
 - named (or atomic) concepts:
Human, Female, Woman...
 - complex concepts / concept expressions:
 \neg **Woman, Human** \sqcap **Female...**
 - (named) basic concepts: **Human, Female...**
 - only used on the r.h.s. of definitions
 - (named) defined concepts: **Woman, Man...**
- acyclicity and
- *unequivocality*, i.e., each named concept used on the l.h.s. of only *a single* definition



Basic and defined concepts and roles

- *Named (or atomic) basic concepts* are given
 - correspond to OWL-NamedClasses that are *not* composed from other classes/properties/...
- *Named (or atomic) defined concepts*
 - correspond to OWL-NamedClasses that are *composed from other classes*
 - defined by *concept expressions*
 - name appears on the left side of an \doteq definition
 - concept expression appears on the right side
- ...similar distinction between *basic and defined roles* later



Roles

- **Mother** \doteq **Female** \sqcap \exists **hasChild**. \top
- **Bachelor** \doteq **Male** \sqcap $\neg\exists$ **hasSpouse**. \top
- **Uncle** \doteq **Male** \sqcap \exists **hasSibling**.**Parent**
 - **roles**: **hasChild**, **hasSibling**...
 - **universal concept** ("top"): \top
 - **existential restriction**: \exists
- **Grandparent** \doteq ..using **Human**, **hasChild**, **Parent**..
- **Grandparent** \doteq ..using only **Human**, **hasChild**..
- **Uncle** \doteq ..using **Male**, **hasSibling**, **hasChild**..



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- **Grandparent** \doteq **Human** \sqcap \exists **hasChild**.**Parent**
- **Grandparent** \doteq **Human** \sqcap
 - \exists **hasChild**. \exists **hasChild**. \top
- **Uncle** \doteq ...using **Male**, **hasSibling**, **hasChild**...



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- **Grandparent** \doteq **Human** \sqcap
 \exists **hasChild**. \exists **hasChild**. \top
- **Uncle** \doteq **Male** \sqcap \exists **hasSibling**. \exists **hasChild**. \top



Null concept

- **Male** \sqcap **Female** \sqsubseteq \perp
 - null concept (“bottom”): \perp
 - subsumption (sub concept): \sqsubseteq
 - equivalence: \equiv
- \doteq is used for *definitions* (or just \equiv)
- \equiv are used for *equivalence axioms*
- \sqsubseteq are used for *subsumption axioms*
 - or: containment / specialisation axioms
- Note the use of $\dots \sqsubseteq \perp$ (“subsumption of bottom”)
 - to say that something is not the case



Null concept

- **Male** \sqcap **Female** $\sqsubseteq \perp$
 - **null concept** (“bottom”): \perp
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 - **equivalence**: \equiv
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 - or: containment / specialisation axioms
- Note the use of $\dots \sqsubseteq \perp$ (“subsumption of bottom”)
 - to say that something is not the case
- *This was our first proper axiom!*
 - so far we have just defined *concepts*
 - we have not used them in proper *axioms*



More uses of roles

- **HappyFather** \doteq **Father** \sqcap
 $\quad \quad \quad \forall$ **hasChild.HappyPerson**
 - universal restriction: \forall
- **MotherOfOne** \doteq **Mother** \sqcap **=1 hasChild.** \top
- **Polygamist** \doteq **≥ 3 hasSpouse.** \top
 - number restrictions: $=, \geq, \leq$
- **Narsissist** \doteq \exists **hasLoveFor.****Self**
 - self references: **Self**
- **MassMurderer** \doteq ...using **hasKilled, Human...**



More uses of roles

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 $\quad \quad \quad \forall$ **hasChild.HappyPerson**
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 - self references: **Self**
- **MassMurderer** \doteq **≥ 4 hasKilled.Human**



Inverse and transitive roles

- **Child** \doteq **Human** \sqcap \exists **hasChild**⁻.**T**
- **hasParent** \doteq **hasChild**⁻
- **hasSibling** \doteq **hasSibling**⁻
- **BlueBlood** \doteq \forall **hasParent**^{*}.**BlueBlood**
 - inverse role: **hasChild**⁻
 - symmetric role: **hasSibling**⁻
 - transitive role: **hasParent**^{*}
- **Niece** \doteq ..**Woman**, **hasChild**, **hasSibling**..



Inverse and transitive roles

- $\text{Child} \doteq \text{Human} \sqcap \exists \text{hasChild}^- . \top$
- $\text{hasParent} \doteq \text{hasChild}^-$
- $\text{hasSibling} \doteq \text{hasSibling}^-$
- $\text{BlueBlood} \doteq \forall \text{hasParent}^* . \text{BlueBlood}$
 - inverse role: hasChild^-
 - symmetric role: hasSibling^-
 - transitive role: hasParent^*
- $\text{Niece} \doteq \text{Woman} \sqcap \exists \text{hasChild}^- . \text{hasSibling} . \top$
- *We just started to define roles!*
 - until now, we have only defined *concepts*



Composite roles

- Similar to composite concepts, e.g.:
 - **hasUncle** \doteq **hasParent** \circ **hasBrother**
 - **hasLovedChild** \doteq **hasChild** \sqcap **hasLoveFor**
 - **hasBrother** \doteq (**hasSibling** | **Male**)
- Mostly *not* supported by reasoning engines
 - they have “bad decision problems”
 - i.e., they compute slowly or intractably
 - ...with some exceptions
- **hasDaughter** \doteq ..using hasChild, Female..



Composite roles

- Similar to composite concepts, e.g.:
 - **hasUncle** \doteq **hasParent** \circ **hasBrother**
 - **hasLovedChild** \doteq **hasChild** \sqcap **hasLoveFor**
 - **hasBrother** \doteq (**hasSibling** | **Male**)
- Mostly *not* supported by reasoning engines
 - they have “bad decision problems”
 - meaning that they compute slowly or intractably
 - ...with some exceptions
- **hasDaughter** \doteq (**hasChild** | **Female**)



TBox

- *Terminology box* (TBox):
 - a collection of axioms and definitions
 - axioms are equivalences or subsumptions:
 - *equivalence axioms* (\equiv):
 - composite concept (role) expressions on both sides
 - *subsumption axioms* (\sqsubseteq):
 - composite concept (role) expressions on both sides
 - terminology boxes can also contain definitions:
 - *definition axioms* (\doteq):
 - defined / named concept (role) on the l.h.s.
 - composite concept (role) expression on the r.h.s
 - make it easier to write other axioms



Acyclic, definitional TBox

Woman \equiv Person \sqcap Female

Man \equiv Person \sqcap \neg Woman

Mother \equiv Woman \sqcap \exists hasChild.Person

Father \equiv Man \sqcap \exists hasChild.Person

Parent \equiv Father \sqcup Mother

Grandmother \equiv Mother \sqcap \exists hasChild.Parent

MotherWithManyChildren \equiv Mother \sqcap ≥ 3 hasChild

MotherWithoutDaughter \equiv Mother \sqcap \forall hasChild. \neg Woman

Wife \equiv Woman \sqcap \exists hasHusband.Man

*Acyclic, and
contains only definitions!*

TBox

- *Acyclic TBoxes:*
 - contains only definitions
 - subsumption axioms can (sometimes) be removed:
 - $T \sqsubseteq C$ is transformed into $T \doteq \bar{T} \sqcap C$
 - Example:
 - $\text{Male} \sqsubseteq \text{Human}$ is transformed into
 - $\text{Male} \doteq \text{Maleness} \sqcap \text{Human}$
 - *when only a single l.h.s. term*
- *Expanded concepts (or roles)*
 - every defined concept (or role) can be *expanded* into an expression of only named basic concepts (or roles)
 - defined only in terms of named *basic concepts* (and *roles*)
 - expanded, definitional TBox



Expanded definitional TBox

*Only named basic concepts
on the right hand sides!*

Woman \equiv Person \sqcap Female

Man \equiv Person $\sqcap \neg(\text{Person} \sqcap \text{Female})$

Mother \equiv (Person \sqcap Female) $\sqcap \exists \text{hasChild}.\text{Person}$

Father \equiv (Person $\sqcap \neg(\text{Person} \sqcap \text{Female})) \sqcap \exists \text{hasChild}.\text{Person}$

Parent \equiv ((Person $\sqcap \neg(\text{Person} \sqcap \text{Female})) \sqcap \exists \text{hasChild}.\text{Person})$
 $\sqcup ((\text{Person} \sqcap \text{Female}) \sqcap \exists \text{hasChild}.\text{Person})$

Grandmother \equiv ((Person \sqcap Female) $\sqcap \exists \text{hasChild}.\text{Person}$)
 $\sqcap \exists \text{hasChild}.\left(\left(\left(\text{Person} \sqcap \neg(\text{Person} \sqcap \text{Female})\right)\right.\right.$
 $\left.\left.\sqcap \exists \text{hasChild}.\text{Person}\right)\right)$
 $\sqcup \left(\left(\text{Person} \sqcap \text{Female}\right)\right.$
 $\left.\sqcap \exists \text{hasChild}.\text{Person}\right)$

MotherWithManyChildren \equiv ((Person \sqcap Female) $\sqcap \exists \text{hasChild}.\text{Person}$) $\sqcap \geq 3 \text{ hasChild}$

MotherWithoutDaughter \equiv ((Person \sqcap Female) $\sqcap \exists \text{hasChild}.\text{Person}$)
 $\sqcap \forall \text{hasChild}.\left(\neg(\text{Person} \sqcap \text{Female})\right)$

Wife \equiv (Person \sqcap Female)
 $\sqcap \exists \text{hasHusband}.\left(\text{Person} \sqcap \neg(\text{Person} \sqcap \text{Female})\right)$

Statements about individuals

- So far axioms about concepts and roles (*TBox*)
- Also two types of axioms about individuals (*ABox*):
 - *class assertion* (using a *concept*):
Märtha : Female \sqcap Royal
 - *role assertion* (using a *role*):
<Märtha, EmmaTallulah> : hasChild
<Märtha, HaakonMagnus> : hasBrother
- *Axioms* about concepts/roles and *assertion axioms* about individuals/roles are used to create knowledge bases:
 - concepts, roles in the *TBox* (aka “the tags”)
 - individuals, roles in the *ABox* (“the tagged data”)



Syntaxes differ a bit...

- So far axioms about concepts and roles (*TBox*)
- Also two types of axioms about individuals (*ABox*):
 - *class assertion* (using a *concept*):
Female(Märtha) , (Female \sqcap Royal) (Märtha)
 - *role assertion* (using a *role*):
hasChild(Märtha, EmmaTallulah)
hasBrother(Märtha, HaakonMagnus)
- *Axioms* about concepts/roles and *assertion axioms* about individuals/roles are used to create knowledge bases:
 - concepts, roles in the *TBox* (aka “the tags”)
 - individuals, roles in the *ABox* (“the tagged data”)



Summary of axioms

- Terminology axioms (in the TBox):

- subsumptions: $C \sqsubseteq D$

C and D are *expressions*,
A is a *defined concept*!

- equivalences: $C \equiv D$

- corresponds to: $C \sqsubseteq D, D \sqsubseteq C$

- definitions: $A \doteq C$

- Individual assertion axioms (in the ABox):

- class assertions: $a : C$

a and b are *individuals*.
R is a *role*!

- role assertions: $\langle a, b \rangle : R$

- A knowledge base $\mathcal{K} = (\mathcal{T}, \mathcal{A})$ consists of

- TBox: \mathcal{T} and ABox: \mathcal{A}



Decision Problems



Reasoning over knowledge bases

- *What more can we do with ontologies?*
- For example:
 - a *security ontology* that describes an organisation and its computer systems as concepts, roles and individuals
 - can answer *competency questions*, e.g.:
 - are all the *security levels* subclasses of one another?
 - what is the highest security level of a *temporary*?
 - what is the necessary security level of a *component*?
 - which employees have access to *critical data*?
 - for which *security roles* is an employee qualified?
 - which individuals are *suspicious persons*?
 - *DL offers a clear and compact way of representing and reasoning about questions such as these!*



Decision problems

- A computational problem with a yes/no answer, e.g.
 - is C *subsumed* by D ($\mathcal{K} \models \mathbf{C} \sqsubseteq \mathbf{D}$)?
 - are C and D *consistent* ($\mathcal{K} \models \mathbf{a} : (\mathbf{C} \sqcap \mathbf{D})$)?
 - does a *belong* to C ($\mathcal{K} \models \mathbf{a} : \mathbf{C}$)?
 - is a *R-related* to b ($\mathcal{K} \models \langle \mathbf{a}, \mathbf{b} \rangle : \mathbf{R}$)?
- *Decidability* (“bestembarhet”):
 - we can always calculate the yes/no answer in finite time
- *Semi-decidability* (“semibestembarhet”):
 - we can always calculate a yes-answer in finite time,
...but not always a no-answer
- *Undecidability* (“ubestembarhet”):
 - we cannot always calculate the answer in finite time

C and D are classes,
a and b are individuals.
R is a role!

Decision problems for concepts

- There are four basic decision problems for concepts:
 - consistency: whether there is an individual **a** so that

$$\mathcal{T} \models \mathbf{a}:\mathbf{C},$$

$$\mathcal{T} \not\models \mathbf{C} \sqsubseteq \perp$$

- subsumption: $\mathcal{T} \models \mathbf{C} \sqsubseteq \mathbf{D},$

$$\mathcal{T} \models \mathbf{C} \sqcap \neg\mathbf{D} \sqsubseteq \perp$$

- equivalence: $\mathcal{T} \models \mathbf{C} \equiv \mathbf{D}$ or $\mathbf{C} \equiv_{\mathcal{T}} \mathbf{D},$

$$\mathcal{T} \models \mathbf{C} \sqsubseteq \mathbf{D}, \mathbf{D} \sqsubseteq \mathbf{C}$$

- disjunction: $\mathcal{T} \models \mathbf{C} \sqcap \mathbf{D} \sqsubseteq \perp$

- *All four can be reduced to subsumption or consistency!*
- \mathcal{T} can be *emptied*, by expanding all its concepts



Decision problems for individuals

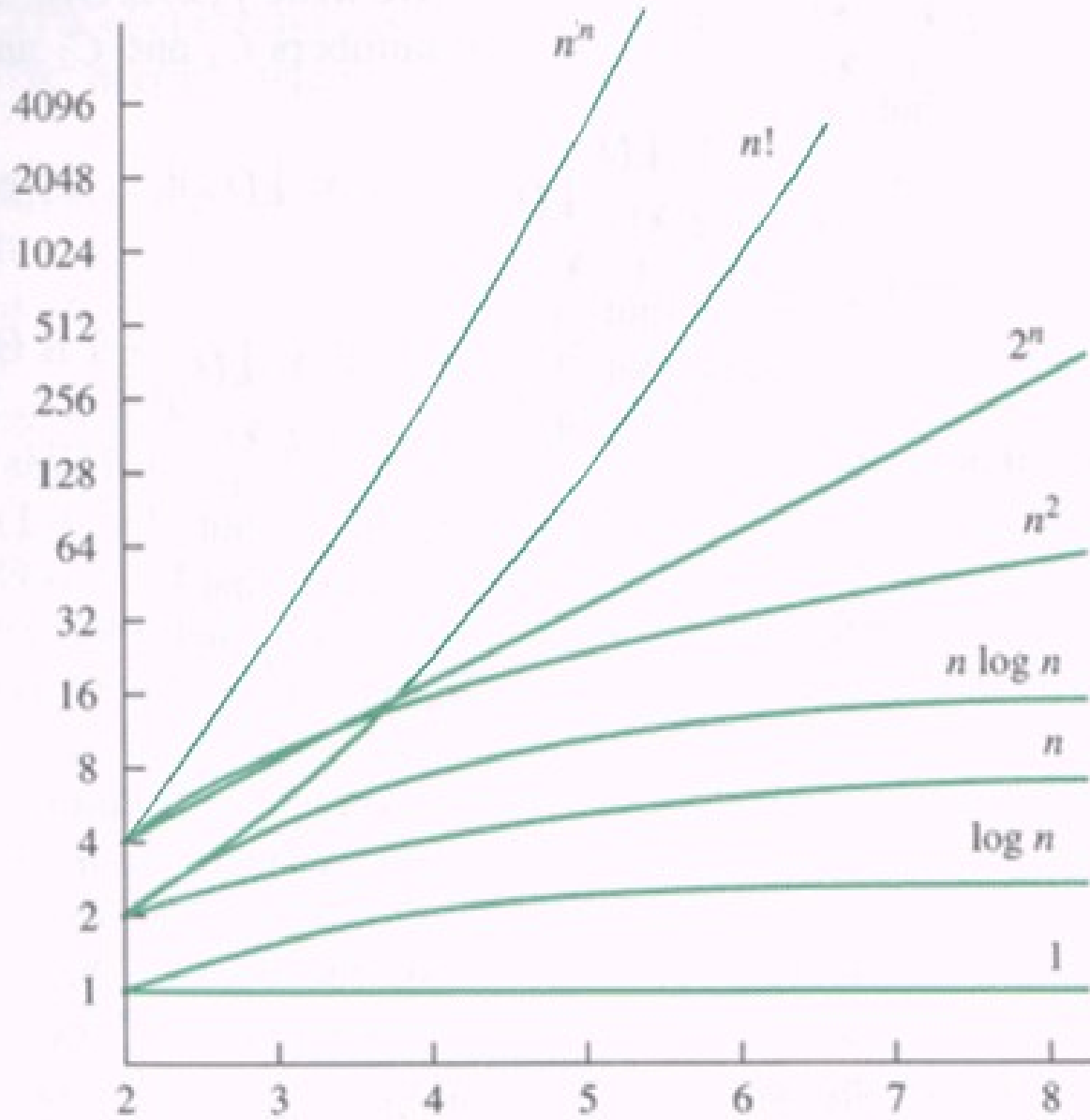
- Decision problems for individuals and roles:
 - instance checking: $\mathcal{A} \models \mathbf{a} : \mathbf{C}$,
 $\not\models \mathcal{A} \sqcap \neg(\mathbf{a} : \mathbf{C})$
is individual \mathbf{a} member of class/concept \mathbf{C} ?
 - role checking: $\mathcal{A} \models \langle \mathbf{a}, \mathbf{b} \rangle : \mathbf{R}$,
 $\not\models \mathcal{A} \sqcap \neg(\langle \mathbf{a}, \mathbf{b} \rangle : \mathbf{R})$
is individual \mathbf{a} \mathbf{R} -related to individual \mathbf{b} ?
 - classifications (not yes/no):
to which classes/concepts does \mathbf{a} belong?
all individuals of class/concept \mathbf{C} ?
- *Everything boils down to consistency checking for ABoxes*
 - ...under certain (rather weak) conditions



Complexity

- Decidability is often necessary
 - but not enough
 - we also want a decision “in reasonable time”
 - different DL-variants have different *complexity*
 - many different *complexity classes*
 - polynomial (**P**), exponential (**EXP**)...
 - ...in time and space
- *Tractable* (or *feasible*) complexity
 - acceptable complexity for large knowledge bases
 - typically *polynomial* complexity (**P**)
 - complexity grows $O(n^c)$ of problem size n





**EXPTIME,
NEXPTIME,
EXPSPACE**

P, NP, PSPACE



DL-complexity

- We have presented many DL-notations
 - *do not* use all at the same time!
 - that gives high complexity
 - which is why we have different OWL Profiles
- Complexity calculator on the net:
 - *Complexity of reasoning in Description Logics*
<http://www.cs.man.ac.uk/~ezolin/dl/>



OWL DL



Relation to OWL

- OWL DL and description logic are closely matched
 - everything in OWL DL has a DL-counterpart
 - most everything in DL can be expressed in OWL DL
- DL is a family of logic systems:
 - some of them correspond to particular OWL profiles
 - OWL1 DL: *SHOIN(D)*
 - OWL2 DL: *SR09Q(D)*



OWL profiles revisited

- **OWL “1”** (2002):
 - OWL Full – “anything goes”
 - OWL DL – fragment of OWL Full,
 - formal semantics through *description logic*
 - OWL Lite – simple fragment of OWL DL, not much used
- **OWL 2** (2008):
 - OWL2 Full – “anything goes”
 - OWL2 DL – fragment of OWL2 full, extension of OWL DL
 - OWL2 EL – quick reasoning, fragment of OWL2 DL
 - OWL2 RL – rule language, fragment of OWL2 DL
 - OWL LD – linked data, fragment of OWL2 RL
 - OWL2 QL – query language, fragment of OWL2 DL

And there is more...

- A few other constructions
- Formal definitions of
 - syntax (rules for valid expressions, reasoning)
 - semantics (rules for interpreting expressions)
- Tools and techniques
- Lots of applications



Manchester OWL syntax



Manchester OWL-syntax

- A simple DL notation without special symbols
 - used by Protege-OWL to construct classes
 - similar to DL syntax
- **Class: Woman**
EquivalentTo: Human and Female
- **Class: Man**
EquivalentTo: Human and not Female
- **Class: Parent**
EquivalentTo: Mother or Father
- Can be used to *serialise* complete ontologies
 - ...we will look mostly at TBox expressions
- <http://www.w3.org/TR/owl2-manchester-syntax/>



Comparison

- DL:

Male \doteq **Human** \sqcap \neg **Female**

- Manchester OWL:

Class: Man

EquivalentTo: Human and not Female

- TURTLE:

family:Man owl:equivalentClass

owl:intersectionOf (

family:Human

[a owl:Class ;

owl:complementOf family:Woman

]

).



Roles in Manchester OWL syntax

- **Class: Mother**
EquivalentTo:
Female and hasChild some owl:Thing
- **Class: Bachelor**
EquivalentTo:
Male and not hasSpouse some owl:Thing
- **Class: Uncle**
EquivalentTo:
Male and hasSibling some Parent
 - universal concept (top): **owl:Thing**
 - existential restriction: **some**



Null concept in Manchester OWL syntax

- **Class:** `<class-name>`
 - EquivalentTo:** `Male and Female`
 - SubClassOf:** `owl:Nothing`
 - null concept (bottom): `owl:Nothing`
 - subsumption (subconcept): **SubClassOf:**
 - equivalence: **EquivalentTo:**
 - ...used both for *definitions* and for *axioms*



More roles in Manchester OWL syntax

- **Class:** HappyFather
EquivalentTo: Father and hasChild **only** Happy
– value restriction: **only**
- **Class:** MotherOfOne
EquivalentTo: Mother and hasChild **exactly** 1
- **Class:** Bigamist
EquivalentTo: hasSpouse **min** 2
– number restriction: **exactly, min, max**
- **Class:** Narcissist
EquivalentTo: loves **some** Self



Inverse, symmetric and transitive roles

- **Class: Child**
EquivalentTo:
Human and inverse hasChild some owl:Thing
- **Class: hasParent**
EquivalentTo: inverse hasChild
- **ObjectProperty: hasSibling**
Characteristic: Symmetric
- **ObjectProperty: hasAncestor**
Characteristic: Transitive
- **inverse role: inverse**
 - **symmetric role:**
Characteristic: SymmetricProperty
 - **transitive role:**
Characteristic: TransitiveProperty

